

Introductory description

A simple approach to data-logging and modelling for physics, using ICT tools with home-produced video clips of students' own dynamic activities. (21 words)

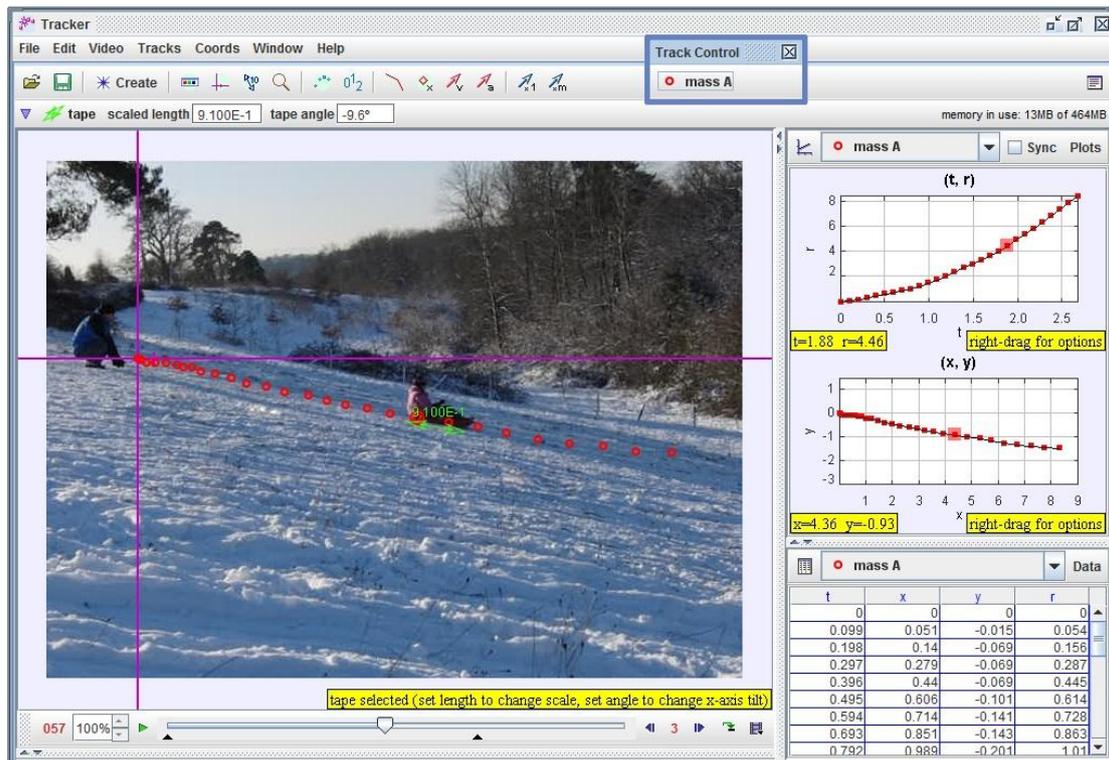
Abstract

Most students now have access to a device (e.g. digital camera or mobile phone) capable of taking short video clips outdoors. Such clips can be used with powerful ICT tools, such as Tracker, Excel and TI-Nspire, to extract time and coordinate data about a moving object, to produce scatter-grams and to fit models. In this article we illustrate the techniques using a video clip of student coming down an icy slope on a toboggan and showing how we can collect enough data to make a good estimate of the coefficient of friction between the toboggan and the ground. (98 words)

Sophie in the snow Adrian Oldknow, Pip Huyton and Ian Galloway

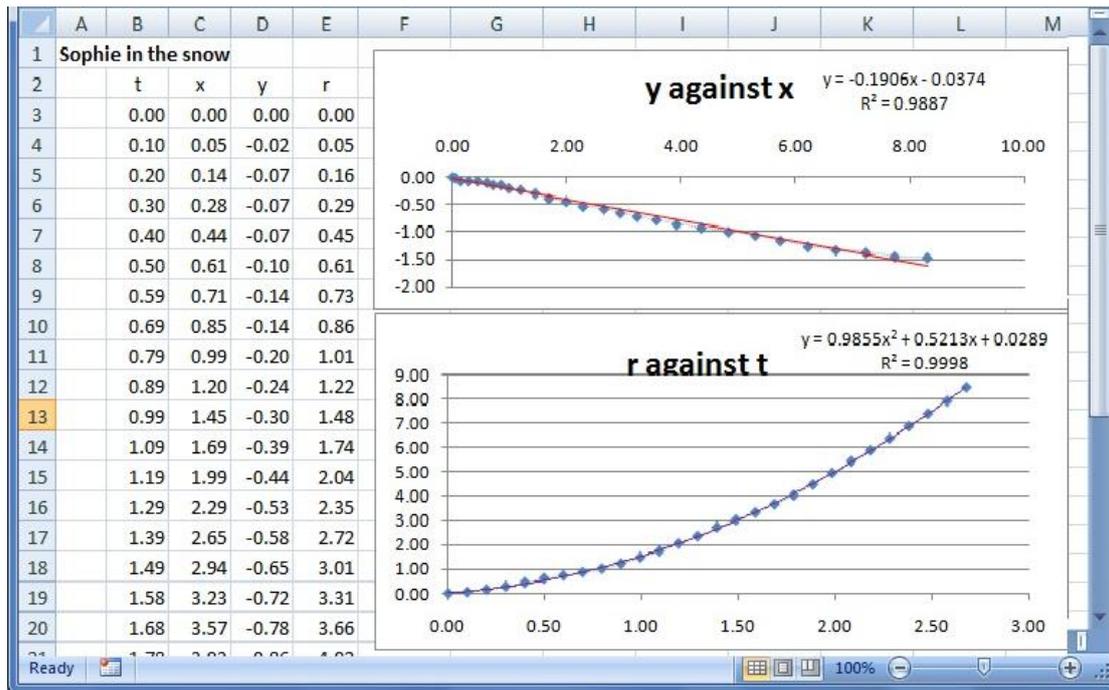
For some time now we have been working with colleagues from maths, science, D&T, ICT and PE/sports on approaches to students' physical activities which provide good opportunities for modelling in maths and science using ICT tools as a catalyst. Here we share just one of the examples to give a flavour of the approach some pilot schools are now pursuing.

Thanks to the inclement weather last December many students were able to take to the slopes with all manner of toboggans. Pip has taken a video of Sophie coming downhill on her red toboggan which is 0.91m long. This was taken on a Casio Exilim digital camera at a frame rate of 30 fps and analysed using the Tracker 3 software – see Figure 1. (Tracker is a free US Open Source Physics Java applet from <http://www.cabrillo.edu/~dbrown/tracker/>). The trick is to keep the camera still (preferably using a tripod) and not to track or zoom to follow the moving object. It is often very helpful to set up an object of known dimensions somewhere near the centre of the scene e.g. a metre rule. Having imported the clip, you specify a known length to calibrate the picture. You can then drag and swing axes to have a suitable origin and direction. Then you can “track” a point on the object while advancing the video clip and so capture data into a table. An advantage of working with video clips taken voluntarily by students and their families away from school is that the activity takes place at their own risk and you do not have to be concerned with a risk assessment!



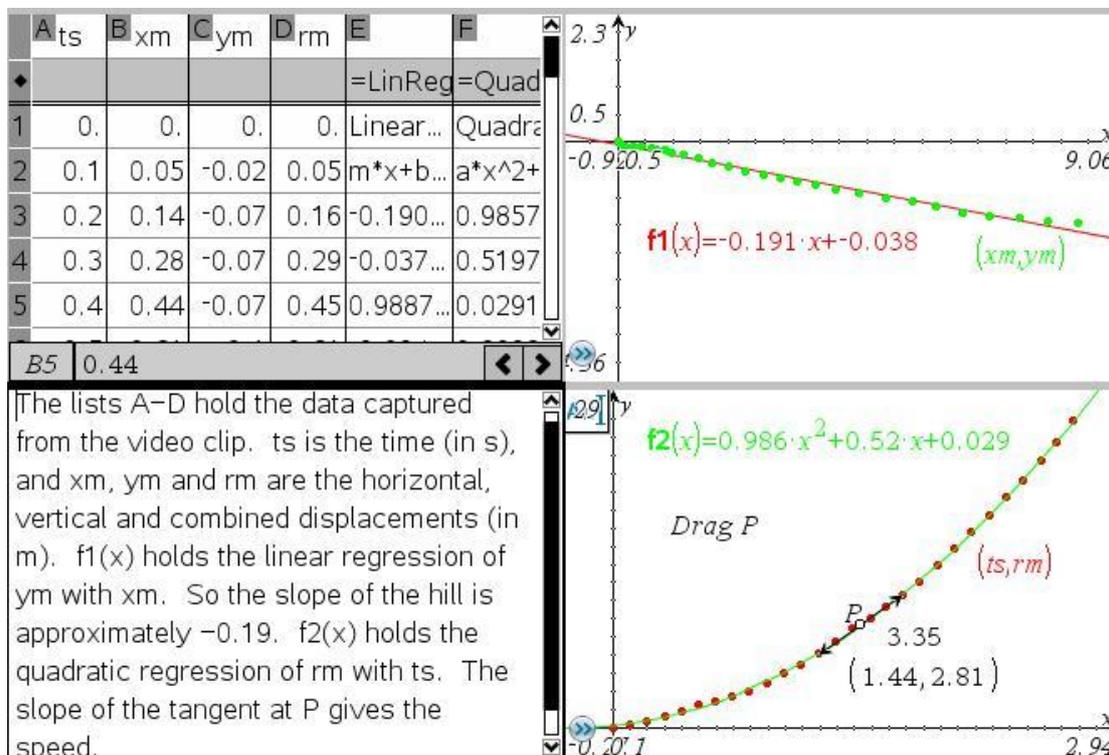
[Figure 1. A screen captured from the Tracker software]

Using the Clip Settings button the video clip has been set to run from frame 0 to frame 80, taking data from every third frame at 30 fps. The auto-tracking has successfully picked up Sophie's foot and the front of the toboggan as target. The x- and y-coordinates are plotted on equal axes in the Plots window to show how the sampled data correspond to the physical hillside. The Tracker variable r measures the displacement of the object from the origin, so we can also show the displacement-time for comparison. We can export the (t, x, y, r) data for further analysis. First we copy and paste the data table to Excel, where we can also display the data, reformat it to 2 decimal places and do some "trend-line" curve fitting, as in Figure 2.



[Figure 2. A screen captured from the MS Excel spreadsheet]

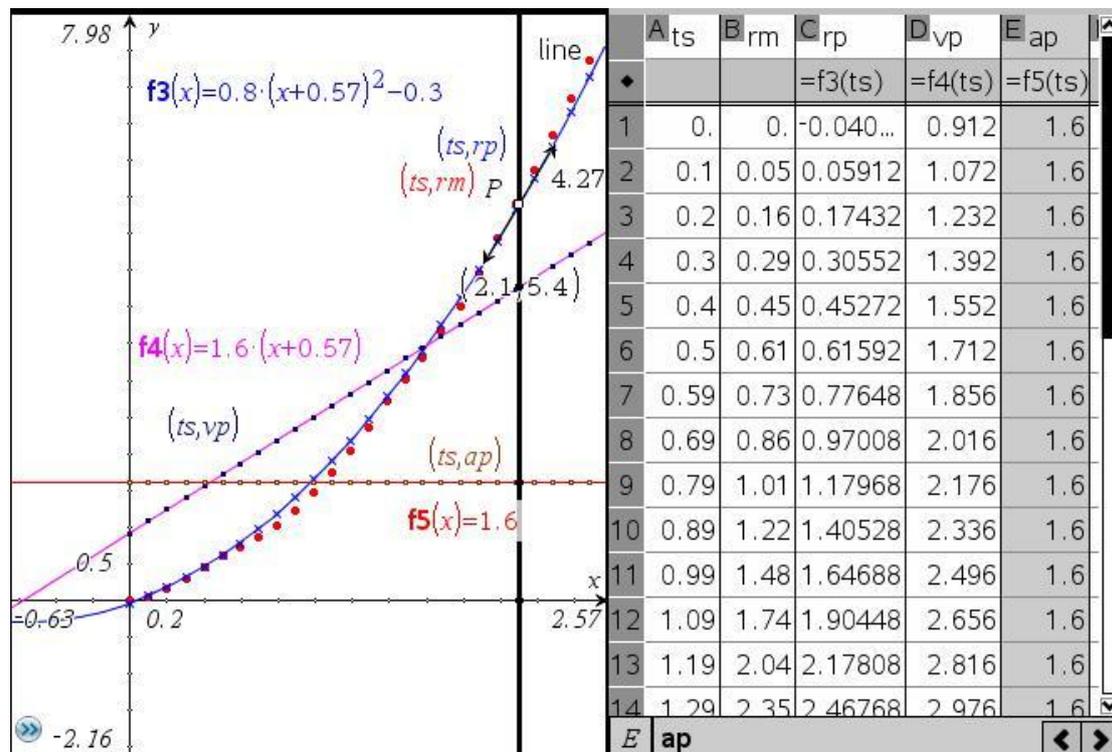
We can then copy and paste the data to the TI-Nspire software for closer analysis. Figure 3 shows a page from a TI-Nspire file designed to be suitable for display on a laptop, or for whole class projection. Exactly the same approach can be used on the handheld TI-Nspire device, but for readability we would normally chose to use pages which each contained less information.



[Figure 3. A page in the TI-Nspire software]

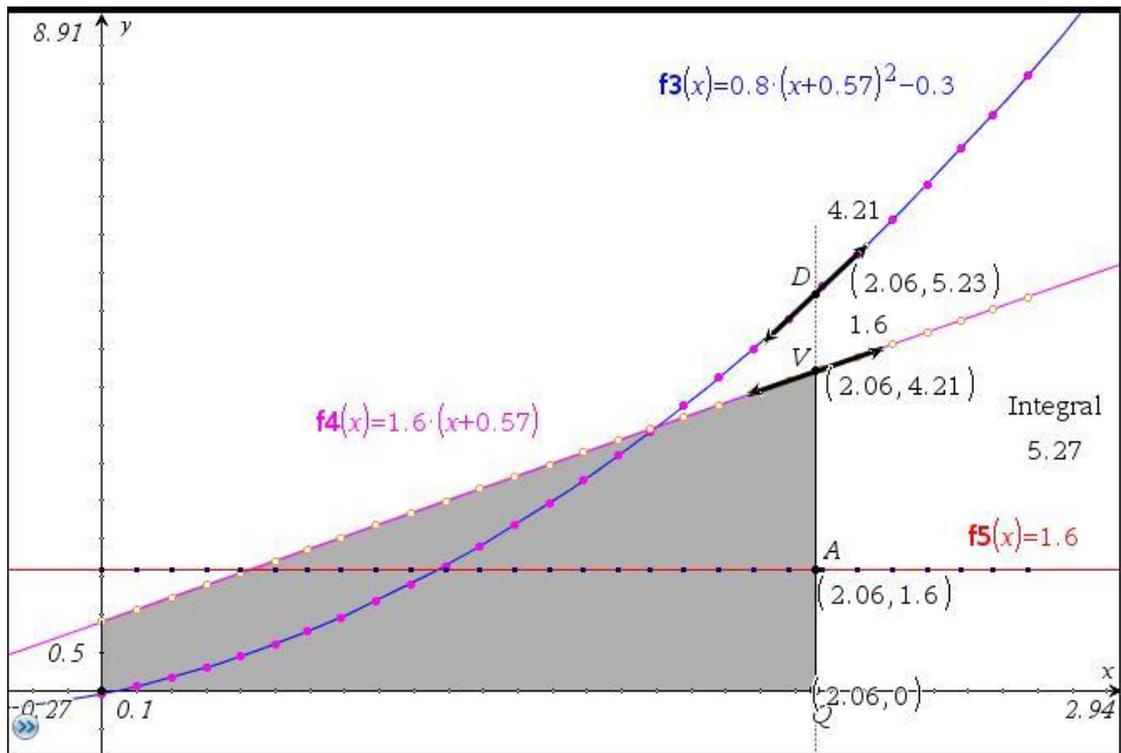
The page is divided between a Lists & Spreadsheets window, two Graphs windows and a Notes window. Here we show for comparison the (xm, ym) scatter plot and its linear regression function – which represents the physical slope of the hillside (and so we need equal intervals on both axes) – together with the displacement data (ts, rm) and its quadratic regression function. The point P has been constructed on the graph, its tangent drawn and its slope measured – so we have a “velocity measurer” for any time t .

Figure 4 shows the next page which is split between a Graphs and a Lists & Spreadsheets window. Instead of using the built-in regression calculations to fit a model to the data we use TI-Nspire’s “flexicurve” approach by first graphing $f3(x) = x^2$, and then dragging the graph’s vertex and flexing its “wings” to change its position and shape – something we will call “drag & flex”.



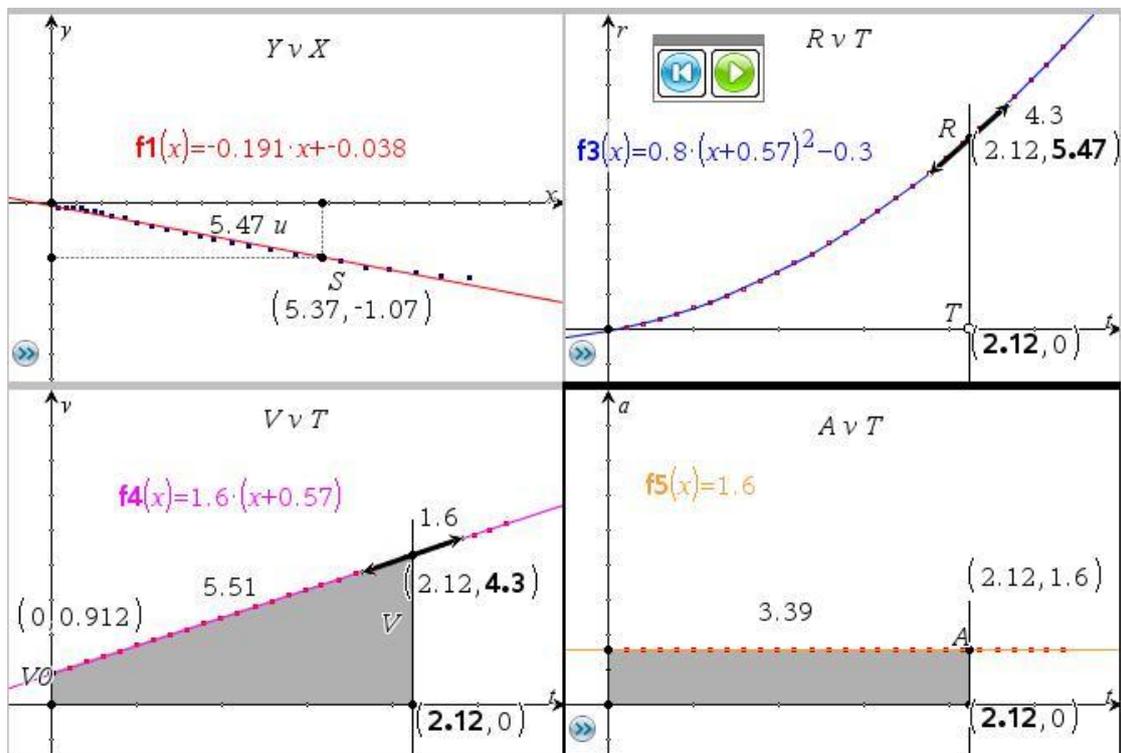
[Figure 4. Modelling data with a flexible curve]

When we have a reasonable “by eye” fit, we can then find the corresponding velocity-time model by simple differentiation of a quadratic polynomial $f3(x)$ to give the linear model $f4(x)$. If we don’t know any elementary calculus we can construct tangents and measure slopes at two or more data points, plot their scattergram, and either make a linear regression, or do a “drag & flex” on the basic linear function $f4(x)=x$. Once we have the linear model $f4(x)$ for velocity we can find the constant model $f5(x)$ for acceleration as the slope of the linear graph. We can calculate predicted values of r , v and a for each value of t , draw the corresponding scatter plots as well as plot the graphs of the model function all on the same axes.



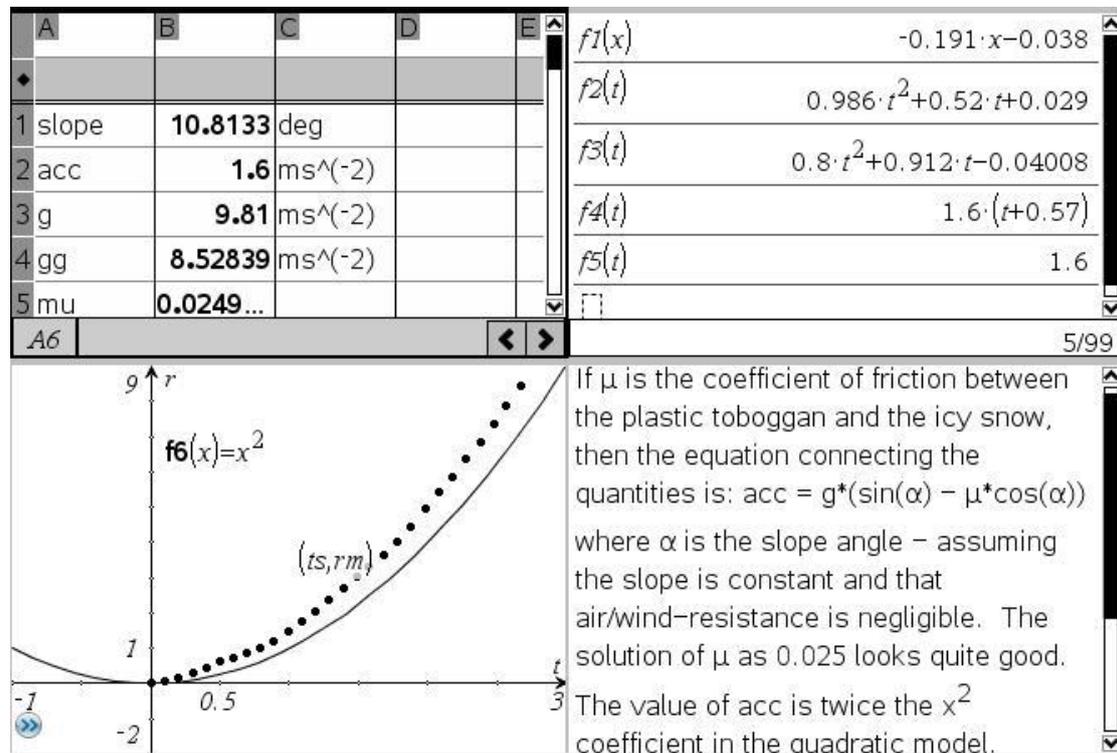
[Figure 5. Moving between graphs]

Figure 5 shows more clearly on a Graphs & Geometry page how to use the tangent/slope technique to find velocity from displacement and acceleration from velocity. Similarly we can see how measuring the area under a graph using the Analyze Integral function enables us to find increase in displacement from velocity, and increase in velocity from acceleration.



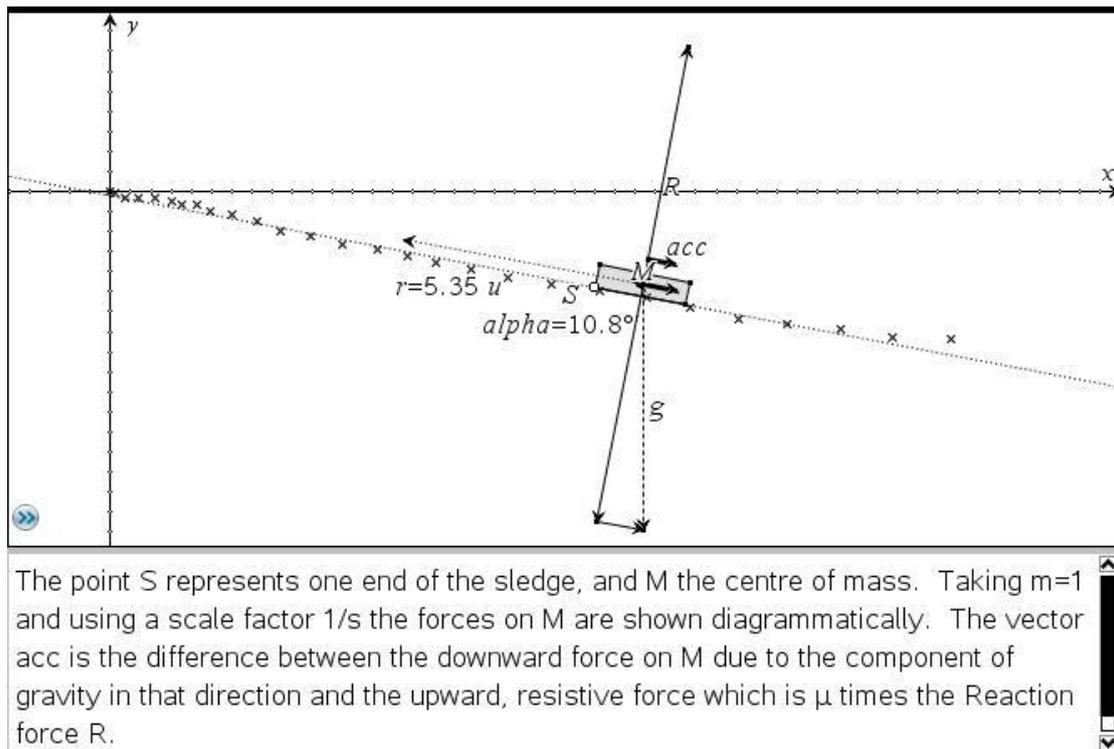
[Figure 6. Linking the graphic representations]

Finally we can bring together the four different graphic representations of the data and their models on the same page, see Figure 6. These are linked together by the motion of the point T on the time axis of the top-right window RvT . You can drag T manually, or make it animate slowly (using the Green arrow). At any time you can see the values of the displacement, velocity and acceleration as well as a simulation of the toboggan coming down the hill!



[Figure 7. The coefficient of friction]

Figure 7 shows a page in which we compute the slope of the hill, which we find to be about a constant 11° downwards. We use it to find an estimate for the coefficient of friction μ between the toboggan's runners and the icy snow. We get a value of 0.025 for μ which agrees well with values from Internet sources suggesting values between 0.015 and 0.04 for sliding on dry snow. However, small changes in the value for the constant acceleration have quite big effects on μ . You can "drag & flex" the quadratic function $f6(x)$ to make a flexi-curve which you can fit using your own "by eye" criteria. We can see then that a value for around μ of 0.03 could be a better bet.



[Figure 8. The force diagram for the toboggan]

We can also use TI-Nspire to illustrate the (scaled) force diagram from which the link between the acceleration and the coefficient of friction is derived – see Figure 8.

In this approach we are, as usual, making many assumptions, such as that the slope is straight, that air-resistance/wind is negligible etc. There is only one real source of possible data error – the calibration of the video. Can you, the reader, find out the effect on calculating μ of, say, a 10% error either way in picking up the 0.91m length of the toboggan in the video? How can we best compensate for perspective distortion of lengths between the centre and the edges of the video frame – or are such effects negligible.

The main point here is that we are not in the realm of absolutes and right answers. This is modelling with real data and, as such, is how mathematics is regularly applied to complex problems. Video capture provides a cheap, portable, reliable and visual means of collecting data in the field. Software tools like Tracker, Vernier's Logger Pro 3, CMA Coach from the Amstel Institute, the Mathematical Toolkit, Dartfish, Silicon Coach, Kandell, Swinger Pro and Quintic all provide the means to gather data from video clips which can be exported to software such as Excel, Fathom and TI-Nspire. What we hope to have shown is how the unique blend of mathematical features contained in TI-Nspire can both help us perform complex calculations supporting mathematical modelling, and also provide means of displays and interactions which enable learners to understand better the close relationship between physical quantities such as displacement, velocity and acceleration.

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